

4.9 (6.4)-Solving Quadratic Equations without Factoring

Vertex form can also be very useful for solving a quadratic equation.

finding roots

Solving Quadratic Equations without Factoring

Recall: To solve by factoring,

- (1) collect all terms on one side of equal sign
- (2) factor the expression
- (3) use $(a)(b) = 0$ to state $a = 0$ or $b = 0$

Consider this example:

$$x^2 - 12x + 32 = 0$$

$$(x - 8)(x - 4) = 0$$

$$x - 8 = 0$$

$$x - 4 = 0$$

$$x = 8$$

$$x = 4$$

(a) Write $y = x^2 - 12x + 32$ in vertex form

$$= (x-4)(x-8)$$

Handwritten notes: $x-4=0 \Rightarrow x=4$, $x-8=0 \Rightarrow x=8$

$$AOS = \frac{4+8}{2}$$

$$= 6$$

$$\text{Sub } x=6$$

$$= (6-4)(6-8)$$

$$= (2)(-2)$$

$$= -4$$

Vertex form

$$y = (x-6)^2 - 4$$

b) Solve for $y = 0$

$$y = x^2 - 12x + 32$$

$$= x^2 - 12x + 36 - 36 + 32$$

$$= (x-6)^2 - 4$$

$$0 = (x-6)^2 - 4$$

$$\sqrt{4} = \sqrt{(x-6)^2}$$

$$\pm 2 = x - 6$$

$$6 \pm 2 = x \rightarrow \begin{cases} x_1 = 8 \\ x_2 = 4 \end{cases}$$

In some cases, one may be simpler than the other.

Ex.2 Write in factored & vertex form, then choose which to use for solving.

(a) $x^2 + 3x - 4 = 0$

$$= (x+4)(x-1)$$

Handwritten notes: $x+4=0 \Rightarrow x=-4$, $x-1=0 \Rightarrow x=1$

$$y = x^2 + 3x - 4$$

$$= x^2 + 3x + 2.25 - 2.25 - 4$$

$$= (x+1.5)^2 - 6.25$$

Ex.2 Write in factored & vertex form, then choose which to use for solving.

(b) $x^2 - 9 = 7$

$$x^2 - 9 - 7 = 0$$

$$x^2 - 16 = 0$$

$$(x+4)(x-4) = 0$$

$$x+4=0 \Rightarrow x=-4$$

$$x-4=0 \Rightarrow x=4$$

$$y = (x-0)^2 - 16$$

If factoring is not possible use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

which is derived from completing the square (p.337-338).

Note: To use the quadratic formula, the equation must be in standard form, $ax^2 + bx + c = 0$.

The ' \pm ' symbol means there are two solutions.

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Ex.3 Solve using the quadratic formula.

a) $x^2 - 4x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{28}}{2} \quad \begin{cases} x_1 = 4.65 \\ x_2 = 0.65 \end{cases}$$

b) $x^2 - 2x - 5 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{24}}{2} \quad \begin{cases} x_1 = 3.45 \\ x_2 = -1.45 \end{cases}$$

Assigned Work:

p.343 # 1ad, 3, 4bdf, 5ace, 9ad, 10d, 14, 19*